

CHARACTERIZATION OF TURBULENT DRAG REDUCTION IN ROTATING DISK SYSTEM

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Abstract—Turbulent drag reduction in a rotating disk apparatus was characterized for a homologous series of polyethylene oxide (PEO) polymers covering a wide range of molecular weights. The concentration dependence of drag reduction in this system was shown to obey an empirical drag reduction equation which had been previously established by Virk et al. for flows in capillary tubes. The linear correlation between polymer concentration and C/DR for different molecular weights of PEO was also obtained. In addition, $[C]$, defined as a characteristic intrinsic concentration, was found to be extremely useful in normalizing the drag reduction data in one homologous series of PEO.

INTRODUCTION

Under certain conditions of turbulent flow, the phenomenon in which the drag of a dilute polymer solution is drastically reduced by even minute amounts of suitable additives has been well-known since Toms [1] first discovered it. This phenomenon implies that polymer solutions undergoing flow in a pipe require a lower pressure gradient to maintain the same flow rate. A higher flow rate would be obtained for the same pressure gradient if a solvent is used. This effect also includes a decrease in the heat transfer coefficient, an alteration in flow patterns around objects, an error in pitot tube measurements, the suppression of cavitation, and a decrease in jet break-up [2]. Therefore, the turbulent drag reduction phenomenon is of considerable engineering interest because of its wide range of application. High molecular weight polymer substances such as synthetics, biopolymers, or surfactants have proved to be effective "drag reducers". Suspended fiber and solid particles may also produce this effect.

Although the mechanism of this drag reduction phenomena has been the subject of extensive research, a complete and satisfactory explanation has not been published as yet. The reason is that an understanding of turbulent drag reduction by dilute polymer

solutions requires the investigation, not only of the phenomena associated with the turbulence itself, but also of the mechanics of macromolecules in a dilute solution. However, several proposed theories are available in these days.

Based on the Oldroyd's theory of slip-at-the-wall [3], Toms [1] originally proposed the idea of a shear-thinning wall layer with an extremely low viscosity. But considering the rheograms of drag reducing polymer solutions, it is evident that they were not shear-thinning. The existence of an anisotropic viscosity, that is a viscosity which is low in the direction of the flow streamlines but high enough in any other direction to dampen a substantial percentage of the oscillation leading to turbulent fluctuations, was thought of as another possible way to explain this phenomenon. Differences in normal stresses could act as a mechanism of drag reduction, but Gadd [4] concluded that there was no obvious correlation between effectiveness in reducing turbulent drag and the differences in these normal stresses. Decreased production of turbulence which was proposed by Gadd [5] is another proposal to explain the drag reduction. Brostow et al. [6] also have recently constructed a theoretical model that explains many of the experimental results concerning drag reduction, based on computer simulations of polymer chain conformations.

In addition, de Gennes [7] introduced an elastic theory of drag reduction to discuss the properties of

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homogeneous, isotropic and three-dimensional turbulence without any wall effect, in the presence of polymer additives. Limited from the start to linear flexible chains in good solvents, the central idea of this 'cascade theory' is that polymer effects at small scales are not described by a viscosity, but by an elastic modulus.

The importance of an elastic property to describe the mechanism of drag reduction was also explained by Armstrong and Jhon [8]. Adopting a simple model to study both the turbulence and dissolved polymer molecules, they related the molecular dissipation to friction factors by constructing a self-consistent method. For the polymer molecules they used a variant of the dumbbell model. The dumbbell representation models a polymer molecule dissolved in a solvent by pretending that it is two spherical (massless) beads connected together by a central spring, immersed in an otherwise Newtonian fluid. In addition, turbulence has been modelled by keeping a kinetic energy budget on the overall flow. They found that a polymer molecule grows by a factor of 10 or more from its equilibrium conformation.

In this study, however, we are not going to further scrutinize the theoretical aspect of the origin of turbulent drag reduction but to find some empirical correlations among molecular parameters of the polymer in our rotating disk system. The rotating disk system can describe the "external flow" that includes the flow over flat plates as well as the flow around submerged bodies. One studies typically the friction drag for an internal flow, whereas one studies the total drag for an external flow. Therefore, in order to study the total drag reduction the rotating disk system is adopted in this paper. It is well known that drag reduction is strongly influenced by these molecular parameters of the dissolved polymer. Therefore, we concentrated our efforts on the characterization of drag reduction of dilute polymer solutions. The concentration dependence of drag reduction was investigated initially. The efficiency of drag-reducing polymer additives, based upon a unit concentration at infinite dilution, was then determined using a characteristic parameter, $DR_{max}/[C]$. Through these analyses, the universal curve established by previous workers [10, 11] for tube flow systems was also obtained for a family of polyethylene oxide in a rotating disk system.

EXPERIMENTAL

Among the drag reducing agents, high molecular weight linear PEO was chosen because of its extreme-

Table 1. Properties of various PEO samples tested

Sample	$\bar{M}_w^{(a)}$ (10^5 g/mole)	$\bar{M}_n$ (10^5 g/mole)
PEO 136E	4	2.9
PEO 343	9	3.7
PEO 344	40	12
PEO 345	50	21.7

^(a)taken from the manufacturer's information

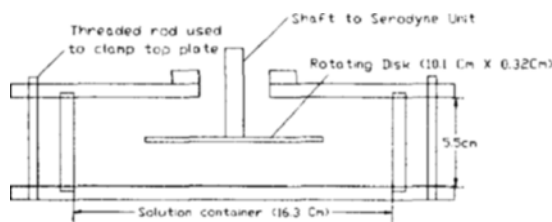


Fig. 1. Side view of the rotating disk device.

ly good solubility in water. Samples were purchased from Scientific Polymer Products Inc. and the weight-average molecular weight of PEO ranged from 4.0×10^5 to 5.0×10^6 g/mole. Molecular weights of the polymers studied are given in Table 1.

Polymer solutions were made by dissolving an appropriate amount of PEO in distilled water. 0.5% (w/w) concentration of stock solutions was initially prepared and then diluted to the required polymer concentration by injecting carefully measured quantities of stock solution directly into the turbulent flow field. 1 wt% isopropanol was also added into a stock solution of PEO to prevent chemical degradation of the polymer.

The rotating disk apparatus was the same as that previously reported [9]. As shown in Fig. 1, it consists of a stainless steel disk whose dimensions are 10.1 cm in diameter $\times$ 0.32 cm in thickness, enclosed in a cylindrical thermostatically controlled container, which is made of stainless steel and whose dimensions are 16.3 cm in ID (inner diameter) $\times$ 5.5 cm height. The whole apparatus was surrounded by a water reservoir whose temperature was well controlled by a constant temperature water circulating system at $(25 \pm 0.5)^\circ\text{C}$. The solution volume required to completely fill the rotating disk apparatus was about 1100 ml. The rotating disk apparatus was rinsed with pure water to prevent any accumulation of polymer into the system. An electric transducer was used to monitor the torque on the disk which was rotated at 2800 rpm, resulting in a water-based Reynolds number of 8.3×10^5 for our system.

Drag reduction measurements were undertaken by injecting measured quantities of stock solution directly

into the turbulent flow field of the rotating disk apparatus. Preliminary flow visualization tests using dyed stock solutions indicated that a complete mixing was accomplished within several seconds. Data were taken by measuring the torque required to rotate the disk at a given speed both in the distilled water (T_0) and in a dilute polymer solution (T_p). The percent drag reduction characteristics were calculated from the following definition;

$$DR = \frac{T_0 - T_p}{T_0} \times 100 \quad (1)$$

where DR is defined to be the percent drag reduction. In addition, an Ubbelohde viscometer was used to measure the intrinsic viscosity $[\eta]$ at $(30 \pm 0.05)^\circ\text{C}$ in deionized water. Viscosity-average molecular weights ($\bar{M}_v$) of PEO were then estimated using the following Mark-Houwink equation;

$$[\eta] = 1.25 \times 10^{-4} \bar{M}_v^{0.78} \quad (\text{at } 30^\circ\text{C}) \quad (2)$$

Eq. (2) is valid for molecular weights of PEO ranged from 2×10^4 to 5×10^6 [12].

RESULTS AND DISCUSSION

To find an empirical function which relates drag reduction to polymer solution properties, Little [10] suggested that the polymer critical concentration might be used to normalize the drag reduction data since the same critical concentration appeared to produce the same degree of drag reduction, irrespective of polymer molecular weight.

Furthermore, Virk et al. [11] experimentally studied the drag reduction caused by dilute PEO solutions in turbulent pipe flow and showed that extent of drag reduction induced by a homologous series of polymers is a universal function of concentration, flow rate, and molecular weight for a given geometry. In developing an empirical relationship between concentration and drag reduction, they defined a characteristic intrinsic concentration, $[C]$, as

$$[C] = \frac{DR_{max}}{[DR]} \quad (3)$$

where C is the polymer concentration, and DR_{max} is the maximum percent drag reduction for a given experimental polymer solution. $[DR]$ is defined as the intrinsic drag reduction:

$$[DR] = \lim_{C \rightarrow 0} \frac{DR}{C} \quad (4)$$

Further, Little et al. [13] introduced a simplified

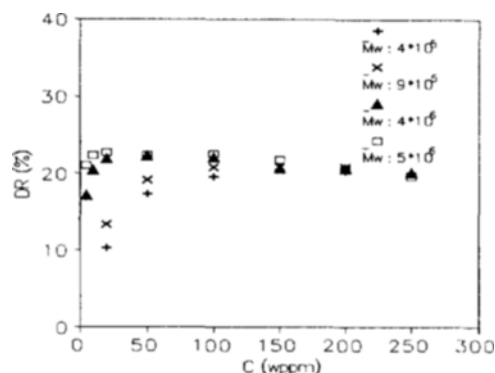


Fig. 2. Drag reduction of PEO versus polymer concentration at various polymer molecular weights.

form of Virk's universal drag reduction relation to account for the concentration dependence of drag reduction in pipe flows. The equation takes the form

$$\frac{(DR/C)}{[DR]} = \frac{1}{1 + C/[C]} \quad (5)$$

at a fixed Reynolds number [13]. The characteristic intrinsic concentration in Eq. (5) was found to be useful in superposing all the experimental data onto a single universal drag reduction curve. This equation indicates that drag reduction becomes more efficient on a unit concentration basis as the concentration decreases. And it also successfully describes the drag reduction results up to concentrations somewhat below that needed to produce an maximum drag reduction. Eq. (3) can be used to simplify Eq. (5)

$$\frac{DR}{DR_{max}} = \frac{C}{C + [C]} \quad (6)$$

The rearrangement of Eq. (6) [using Eq. (3)] leads to

$$\frac{C}{DR} = -\frac{[C]}{DR_{max}} \times \frac{C}{DR_{max}} \quad (7)$$

Eq. (7) shows that there is a linear relationship between C/DR and C when the concentration dependence of drag reduction obeys Eq. (6). This drag reduction equation has been found to be applicable to most drag reducing polymers [13, 14].

Therefore, we find a linear relationship between C/DR and C from the above equation. In our previous paper [9], we studied the effect of concentration on the drag reduction and showed the existence of a critical concentration at which the drag reduction was maximized.

Fig. 2 shows the dependence of percent drag reduc-

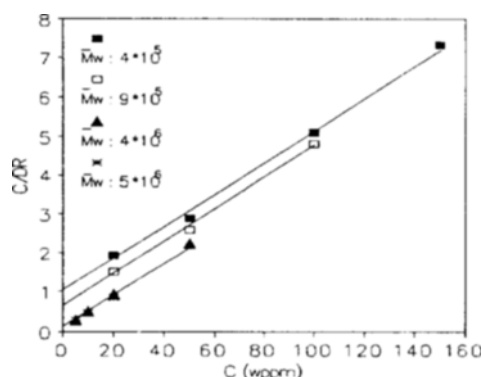


Fig. 3. Concentration dependence of drag reduction for PEO.

tion on four different molecular weights of PEO as a function of polymer concentration up to 250 wppm (ppm based on weight) at 2800 rpm for the rotating disk system. Values of maximum drag reduction of different molecular weights of PEO are obtained from this experiment and further used for the analysis in this paper. Those are 20.5% for PEO 136E, 20.8% for PEO 343, 22.3% for PEO 344 and 22.7% for PEO 345. It indicates clearly that the concentration required for maximum drag reduction decreases with increasing molecular weight. On the other hand, it also shows that the molecular weight dependence of drag reduction becomes negligible above about 150 wppm of PEO concentration.

The linear correlation between polymer concentration and C/DR for four different molecular weights in a range of conditions close to the maximum drag reduction is illustrated in Fig. 3. Only dilute solution behavior is considered. Little et al. [13] explained that Henry's law is applicable in this range, in which $C/[C]$ values approach 0.01. For PEO having a molecular weight of 7×10^6 the distance between the random coils of the polymers at this level of drag reduction is 20 diameters. The parameter $DR_{max}/[C]$, the Henry's law constant, shown in Eq. (8) defines the efficiency of the polymer additives on a unit concentration basis at infinite dilution:

$$\lim_{C \rightarrow 0} \frac{DR}{C} = \lim_{C \rightarrow 0} \frac{DR_{max}}{C + [C]} = \frac{DR_{max}}{[C]}. \quad (8)$$

Only at these intermolecular distances is the percent drag reduction a linear function of polymer concentration. Near the maximum DR condition, the polymer molecules are only a few diameters apart and virtually touch. Eq. (3) is valid not only in turbulent pipe system but also our turbulent rotating disk sys-

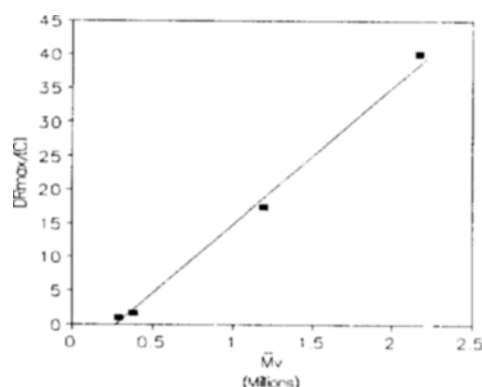


Fig. 4. Plot of $DR_{max}/[C]$ versus molecular weight for PEO.

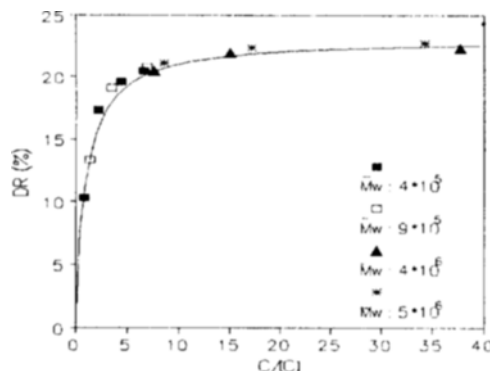


Fig. 5. Intrinsic concentration as a normalizing factor for PEO.

tem. The intercept value at $C/DR=0$ yields the intrinsic concentration $[C]$ and this quantity divided by the intercept at $C=0$ permits evaluation of DR_{max} .

Fig. 4 shows a correlation between the intrinsic drag reduction in Eq. (8) and the viscosity average molecular weight for the homologous series of PEO. The plot is linear and the intercept value of $\bar{M}_v = 2.65 \times 10^5$ obtained from the least square method gives a cutoff point in molecular weight below which no drag reduction takes place at this Reynolds number of 8.3×10^5 . This value is comparable with that obtained for the Polyox family (2.46×10^5) in pipe flow system [15].

Furthermore, $[C]$ was found to be extremely useful in normalizing the drag reduction data of different molecular weight compounds into one homologous series as shown in Fig. 5. This figure demonstrates that the intrinsic concentration can be used to normalize data obtained from our rotating disk system. On the other hand, if we define $(DR/C)/[DR]$ as β and $C/[C]$ as α , Eq. (5) becomes $\beta = 1/(\alpha + 1)$. This figure also

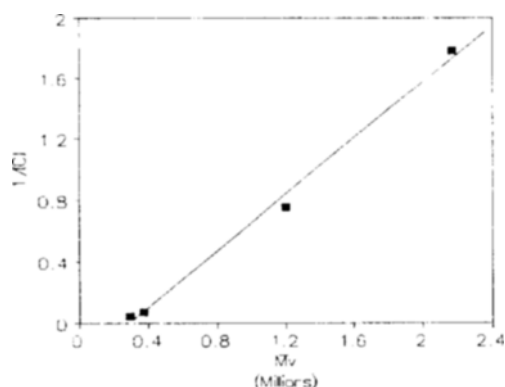


Fig. 6. Plot of $1/[C]$ versus viscosity-average molecular weight for PEO.

indirectly indicates that Eq. (5) is a universal curve equation for the PEO family and works well for the rotating disk system. However, for toluene-soluble polyisobutylenes (PIB), Dschagarowa and Mennig [16] obtained the universal curve $\beta = 1/(\alpha + 0.4)$. From the similar behavior observed for both water-soluble PEO and toluene-soluble PIB, they proposed a universal curve with a single parameter which depends on polymer/solvent system.

$$\beta = 1/(\alpha + K). \quad (9)$$

The constant K is, therefore, characteristic of a particular polymer family in a given solvent and does not depend on the molecular weight or flow geometry. In addition, normalizing the drag reduction data of four PEO compounds using $[C]$, we find that the reciprocal of $[C]$ is a linear function of the viscosity-average molecular weight as shown in Fig. 6. The linear relationship is also represented by the following equation:

$$\bar{M}_v \times 10^{-6} = \frac{1.23}{[C]} + 0.25. \quad (10)$$

$\bar{M}_v/DR_{max}$ is also correlated with the viscosity-average molecular weight as shown in Fig. 7 and can be approximated with

$$DR_{max} = \frac{23.7 \bar{M}_v}{\bar{M}_v + (2.61 \times 10^4)}. \quad (11)$$

As shown in Fig. 2, DR_{max} grows within the investigated molecular weight range, while the required concentration decreases with increasing molecular weight of polymers of the same homologous series. From the Einstein equation, which shows that equivalent sphere density is inversely proportional to coil dimensions,

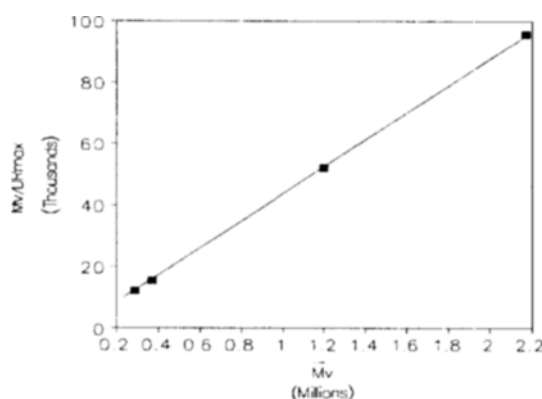


Fig. 7. Plot of $\bar{M}_v/DR_{max}$ versus viscosity-average molecular weight.

it can be further concluded that as the intrinsic viscosity of homologous series of PEO in the rotating disk system increases, the dimensions of their coils in solution and their DR efficiency increase.

On the other hand, Little et al. [13] showed graphically that $1/[C]$ is a linear function of $1/C_c$, where C_c is the critical concentration for the touching of polymer random coils. From their relationship, a correlation between the intrinsic concentration and the intrinsic viscosity was developed for PEO in water for their system:

$$[C] ([\eta] - 2.99) = 21.9 \times 10^4 \quad (12)$$

From Eq. (12), one observes a linear relationship between $1/[C]$ and $[\eta]$.

Through this study, we can thus conclude that the empirical equations to relate drag reduction to relevant solution properties would reduce the amount of time spent in evaluation procedures. The polymer concentration was used to normalize the drag reduction data since the same fraction of critical concentration appeared to produce the same degree of drag reduction, irrespective of polymer molecular weight.

CONCLUSIONS

In this study, the effects of concentration and molecular weight for a homologous series of PEO on drag reduction were investigated in a rotating disk apparatus. The concentration required for DR_{max} was found to decrease with increasing molecular weight and the molecular weight dependence of DR becomes negligible above about 150 wppm of PEO concentration.

Linear correlations between polymer concentration and C/DR , drag reduction index and viscosity-average

molecular weight, $1/[C]$ and $\bar{M}_v$, and $\bar{M}_v/DR_{max}$ and $\bar{M}_v$ are also found in this rotating disk system. In addition, $[C]$ was found to be extremely useful in normalizing the drag reduction data of different molecular weight compounds into one homologous series of PEO.

When $(DR/C)/[DR]$ and $C/[C]$ are defined as β and α , respectively, it was possible to obtain a universal curve represented by $\beta = 1/(\alpha + 1)$ for PEO family. This result suggests that a general universal curve can be written as $\beta = 1/(\alpha + K)$, where the constant K is characteristic of a particular polymer family in a given solvent and does not depend on the molecular weight or flow geometry.

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